

# Set Functions for Time Series

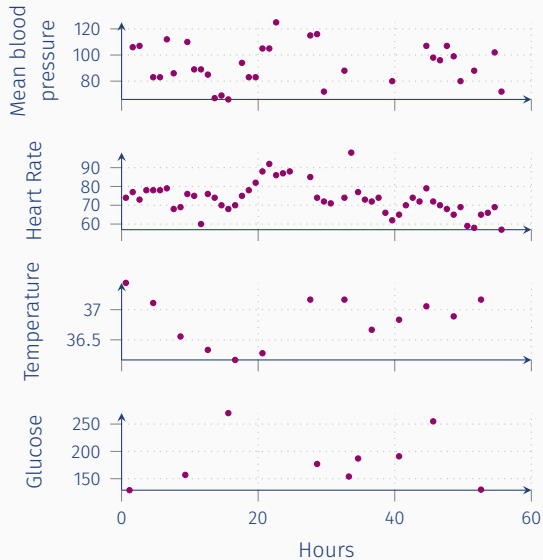
ICML 2020

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**Max Horn**, Michael Moor, Christian Bock, Bastian Rieck and Karsten Borgwardt

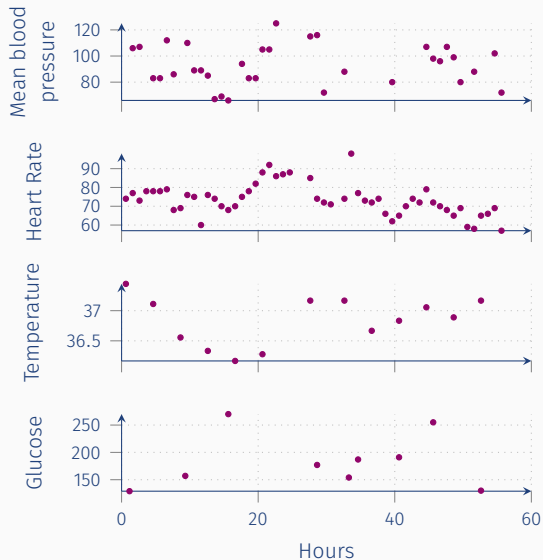
Machine Learning and Computational Biology Group, ETH Zurich

# Motivation - Medical time series



## Challenges

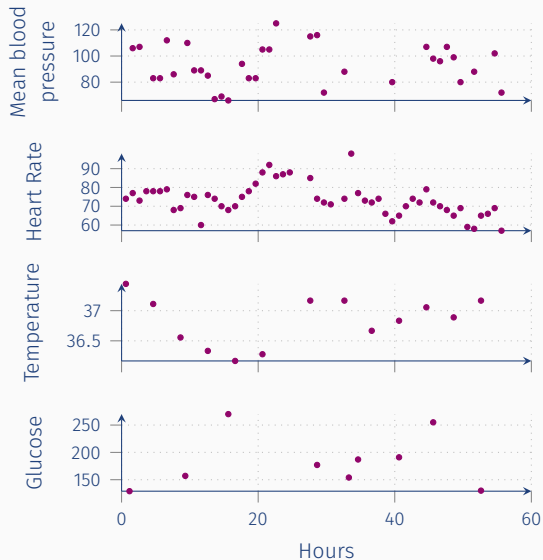
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## Challenges

- Irregular sampling of data

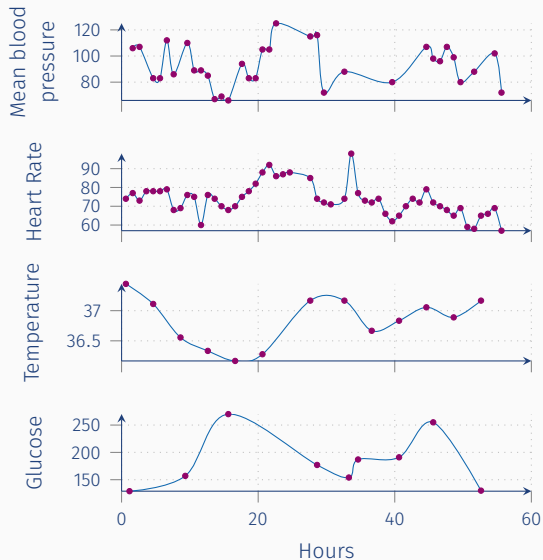
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## Challenges

- Irregular sampling of data
- High demands on interpretability

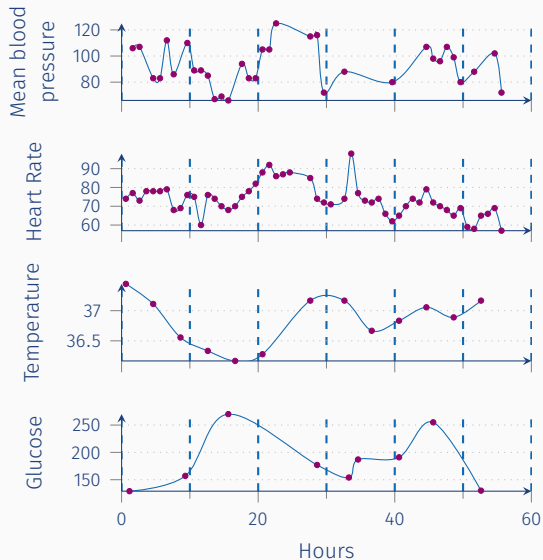
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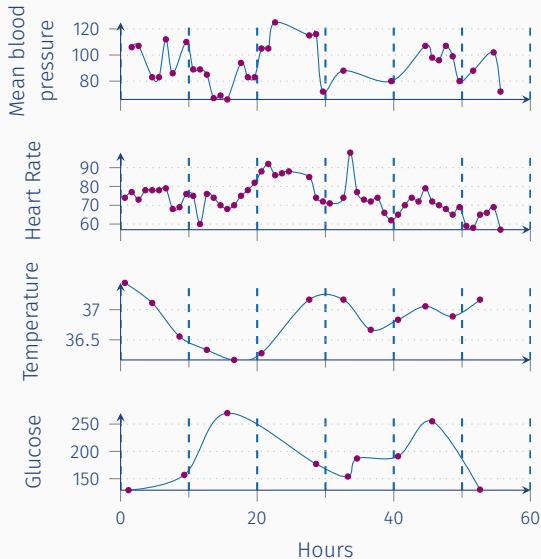
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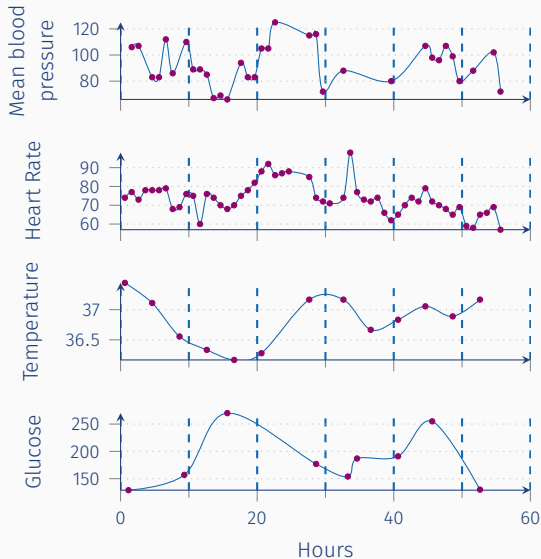
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- Irregular sampling of data
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## Problem statement

Learning classification models on irregularly-sampled time series without prior imputation.

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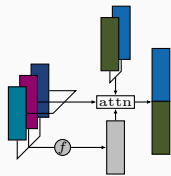
Learning classification models on irregularly-sampled time series without prior imputation.

## Set Functions for Time Series

→ Time series classification as set classification

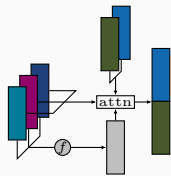




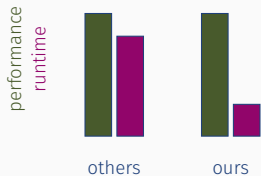


New approach for  
Irregularly-sampled Time  
Series

# Contributions

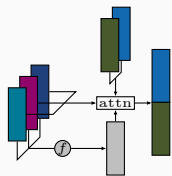


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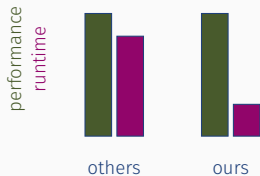


Competitive Performance  
with Lower Runtime

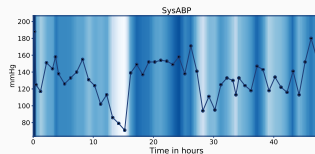
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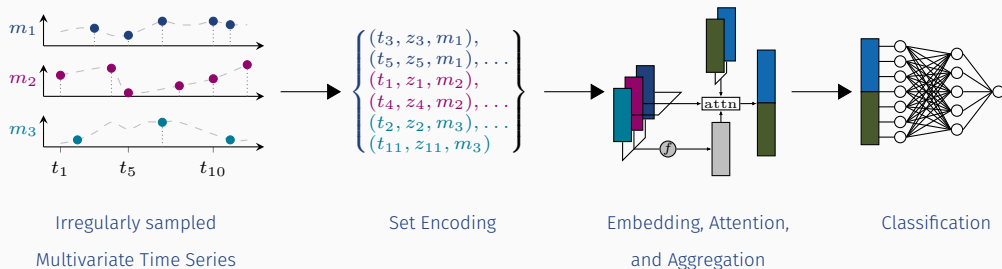


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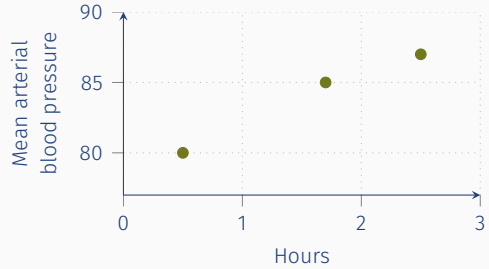
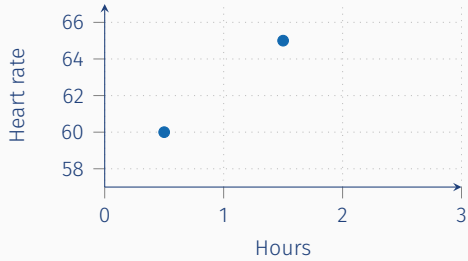


Per Observation  
Contributions

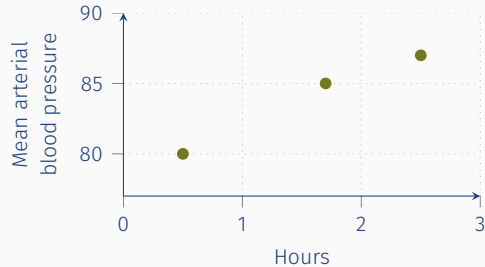
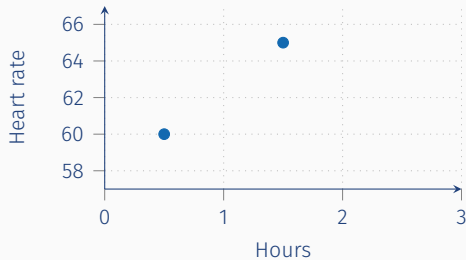
# Architecture Overview



# Key idea - Time Series as Sets of Observations

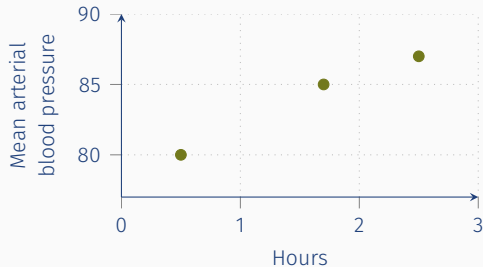
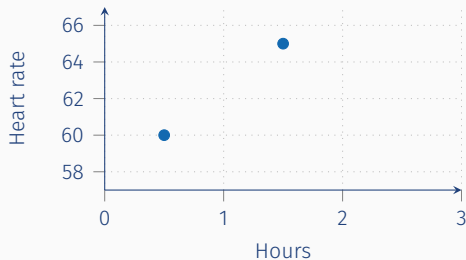


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Each observation  $s_j$  is represented as a tuple  $(t_j, z_j, m_j)$

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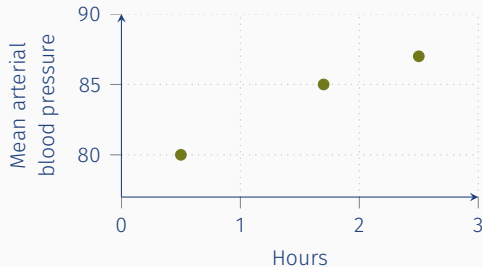
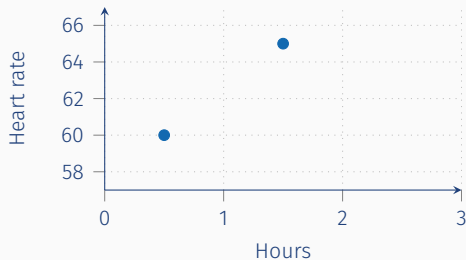


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Each observation  $s_j$  is represented as a tuple  $(t_j, z_j, m_j)$

$$\mathcal{S} = \{(0.5, 60, 1), (1.5, 65, 1), (0.5, 80, 2), (1.7, 85, 2), (2.5, 87, 2)\}$$

$$f(\mathcal{S}) = g \left( \frac{1}{|\mathcal{S}|} \sum_{s_j \in \mathcal{S}} h(s_j) \right)$$

where  $h: \Omega \rightarrow \mathbb{R}^d$  and  $g: \mathbb{R}^d \rightarrow \mathbb{R}^c$  are neural networks

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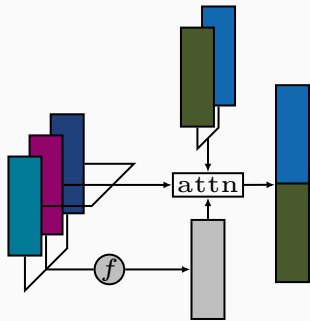
## Problem

Influence of an element shrinks as  $|\mathcal{S}|$  grows!

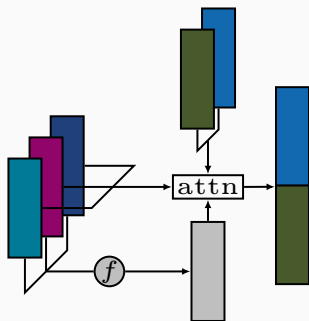
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# Set Attention Mechanism

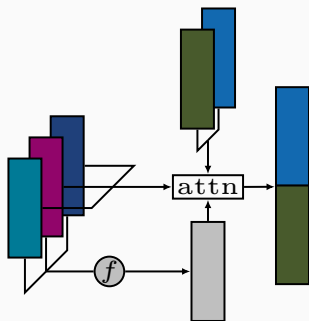


# Set Attention Mechanism



Keys:  $K_{j,i} = [f(\mathcal{S}), s_j]^T W_i$

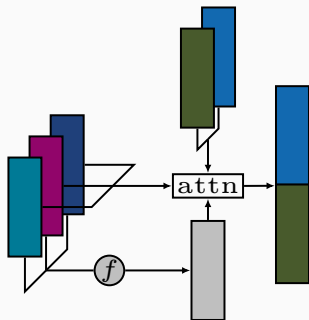
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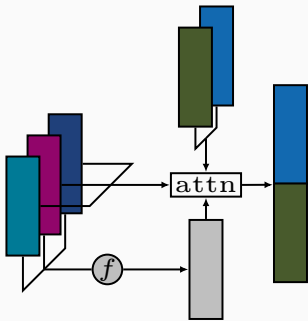


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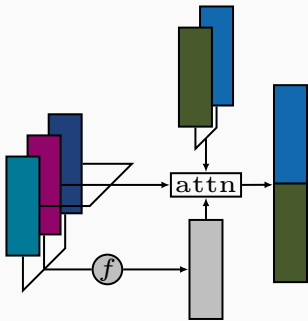
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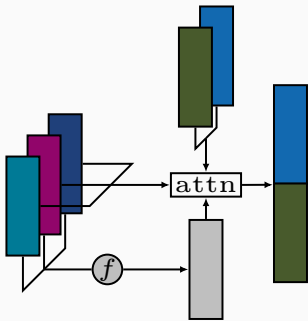
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$$\mathcal{L}(\theta, \psi) = \mathbb{E}_{(\mathcal{S}, y) \in \mathcal{D}} \left[ \ell \left( y; g_\psi \left( \sum_{s_j \in \mathcal{S}} a(\mathcal{S}, s_j) h_\theta(s_j) \right) \right) \right]$$

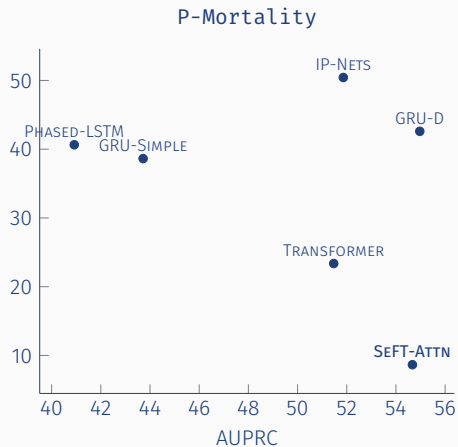
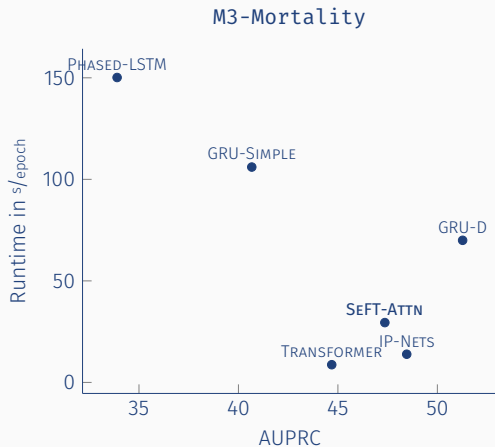
## Datasets

- Two mortality prediction tasks - MIMIC-III (**M3-Mortality**) and Physionet 2012 (**P-Mortality**)
- Sepsis early recognition task - Physionet 2019 Challenge

## Comparison partners

- PHASED-LSTM – *Neil et al., NeurIPS 2017*
- TRANSFORMER – *Vaswani et al., NeurIPS 2017*
- GRU-SIMPLE & GRU-D – *Che et al., Scientific reports 2018*
- IP-NETS – *Shukla & Marlin, ICLR 2019*

# Results - Performance vs. Runtime



## Results - Sepsis Early Prediction

| Model       | B-Accuracy   | AUPRC        | $U_{\text{norm}}$ | $s/\text{epoch}$ |
|-------------|--------------|--------------|-------------------|------------------|
| GRU-D       | 51.15        | 5.82         | 0.02121           | 190.41           |
| GRU-SIMPLE  | 50.69        | 6.97         | 0.01309           | 92.90            |
| IP-NETS     | <b>78.02</b> | 37.60        | <b>0.51327</b>    | 232.92           |
| PHASED-LSTM | 50.09        | 6.40         | 0.00159           | 110.49           |
| TRANSFORMER | 77.84        | <b>55.30</b> | 0.49974           | 71.70            |
| SEFT-Attn   | 74.50        | 8.78         | 0.34120           | <b>62.91</b>     |

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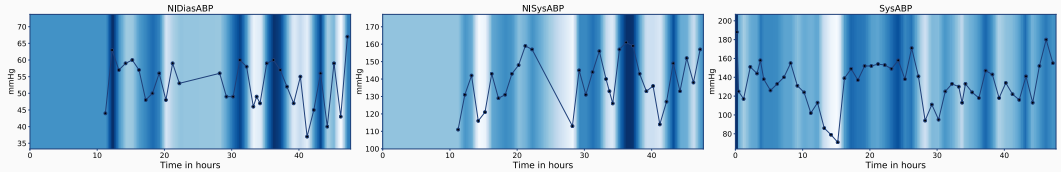
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### Possible Leakage of Future Information

**IP-NETS** Through unmasked interpolation

**TRANSFORMER** Through layer normalization

# Results - Interpretability

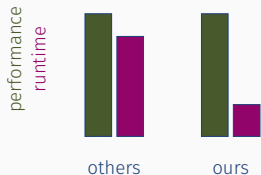


Uniquely allows a **per-observation** quantification of importance

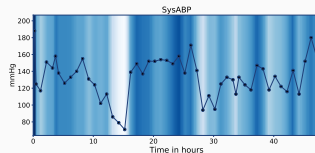
# Summary

$$\left\{ \begin{array}{l} (t_3, z_3, m_1), \\ (t_5, z_5, m_1), \dots \\ (t_1, z_1, m_2), \\ (t_4, z_4, m_2), \dots \\ (t_2, z_2, m_3), \dots \\ (t_{11}, z_{11}, m_3) \end{array} \right\}$$

New approach for  
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Competitive performance  
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Per observation  
contributions

For further information please check out our paper.

